Gaussian

$$\mathscr{P}(x)dx = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2} dx \quad ; \mathscr{P}(x)\text{는 가우스형태의 확률분포}$$

규격화

$$\int_{-\infty}^{\infty} \mathscr{P}(x)dx = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} e^{-(x-\mu)^2/2\sigma^2} dx$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \sqrt{2\sigma^2 \pi} = 1$$

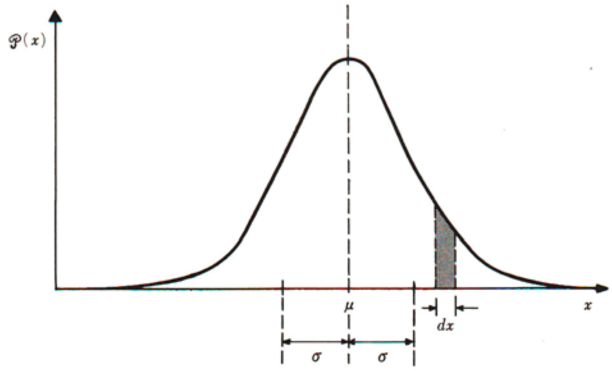

Fig. 1·6·2 The Gaussian distribution. Here $\mathscr{P}(x)\,dx$ is the area under the curve in the interval between x and $x + dx$ and is thus the probability that the variable x lies in this range.

x의 평균값

$$\overline{x} = \int_{-\infty}^{\infty} x\mathscr{P}(x)dx = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} x e^{-(x-\mu)^2/2\sigma^2} dx \quad (\, y = x-\mu \rightarrow dx = dy \,)$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} (y+\mu) e^{-\frac{y^2}{2\sigma^2}} dy = \mu$$

분산

$$\overline{(\Delta x^2)} = \overline{(x-\overline{x})^2} = \overline{x^2} - \overline{x}^2 = \overline{x^2} - \mu^2$$

$$\overline{(x-\mu)^2} = \frac{1}{\sqrt{2\pi}\,\sigma} \int (x-\mu)^2 e^{-(x-\mu)^2/2\sigma^2} dx$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int y^2 e^{-\frac{y^2}{2\sigma^2}} dy$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \left[\frac{\sqrt{\pi}}{2} (2\sigma^2)^{\frac{3}{2}} \right]$$

$$= \sigma^2$$

$$\overline{(\Delta x)^2} = \overline{(x-\mu)^2} = \sigma^2$$